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TECHNICAL MEMORANDUM

"DETECTION" AND "FALSE ALARM" PROBABILITIES FOR VARIOUS DISPLAY MARKING CRITERIA

by
H. C. Boehme

Submitted to

Commander
Naval Ship Systems Command
Department of the Navy
Washington, D. C. 20360
Attention: Code PMS-87

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TABLE OF CONTENTS

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"DETECTION" AND "FALSE ALARM" PROBABILITIES FOR VARIOUS DISPLAY MARKING CRITERIA

1. INTRODUCTION

In an automated detection system (such as a simulation model) decision rules must be specified. The resulting model predictions are critically dependent upon these decision rules or criteria. If one hopes to match operator performance over a wide range of conditions using an automated detection system, the criteria should be made as sensitive to the operator's stimuli as possible. The adequacy of correlation between experimental results and model predictions will depend upon the specific criteria chosen to represent the human operator's function in the simulation model.

A visual display, for example, provides information to an operator regarding the presence of signal-plus-noise as opposed to noise alone according to the number, arrangement or pattern, and intensity of marks. If the operator's decision rule (or rules) were known it would be possible to predict his performance provided that one could accurately represent the hardware portion of the system.

The simulation model of a total system can be designed in modular form with each module representing the transfer characteristics of a specific component. In many applications total system performance is predicted by simulation of the system up to the output of the signal processors. "Probability of detection" generally refers to the probability that an independent sample of the processed signal-plus-noise at the processor output exceeds a specified threshold. The rate at which independent samples of the processed noise alone exceed the same threshold determines the "false alarm rate."

No serious confusion exists in using the terms above when comparing the outputs of different processors even though detection and false alarm are specific operator functions. When different signal processors form parts of more complex systems and predictions of system performance are sought, the above terminology may be misleading and, indeed, the above comparison itself may be inadequate.

Signal processors may drive displays which are subsequently viewed by human operators who must respond to the visual stimuli. In this case comparison of total system performances, including operators, may be more meaningful. This is especially true when operator response may significantly affect system performance.

In its simplest form the representation of an operator interacting with a visual display may be a set of decision rules. With this representation the "probability of detection" is the probability that signal-plus-noise will cause the display to be marked in such a way that the decision rule or operator criterion is satisfied. "False alarm rate" is the rate at which noise alone marks the display such as to satisfy the same criterion. The purpose of this report is to outline the computational requirements for detection and false alarm probabilities for various display marking criteria.

The probability of detection, P(D), and probability of false alarm, P(FA), as used in this outline refer only to single display probabilities and no attempt has been made to include interrelationships between separate displays. In describing the manner in which the single ping probabilities are obtained, a display matched to the output of the processor is used. In the event of a mismatched display and processor, curves describing the inputs to the display would be required. A uniform noise marking density has also been assumed for simplicity.

2. SUMMARY

False alarm and detection probability statements are presented for various display marking criteria. These statements are given in terms of the marking probabilities for single range resolution intervals of the display. Intensity and pattern of marks are included in the criteria and a procedure is outlined for obtaining single range resolution interval probabilities from signal processor curves in the case of a matched display. Both zero and non-zero range rate targets are considered.

The case of a low noise marking density is considered in some detail. Physical arguments are then presented for reducing to a more tractable approximate form the expressions for high marking densities. The arguments lead to estimates of the error introduced by use of approximate or other than exact expressions.

3. THREE PING CYCLE HISTORY - ZERO RANGE RATE

The calculation of the probability of detection, P(D), and the probability of false alarm, P(FA), will be considered for a multi-ping history which meets the following conditions:

1. Three ping cycle history displayed - all three erased at the end of the third ping cycle.

2. Continuous intensity levels of marks approximated by 7 discrete intensity levels.

3. Probability of a noise mark at minimum intensity level I_j in any resolution interval is p_j and is constant for the three-ping history.*

4. The noise marking density is low enough to ignore the possibility of two marks occurring in a range interval which includes five resolution intervals on any ping cycle.

The following definitions will be used:

A. The detection criterion, $[s \geq \lambda;\ I \geq I_j]$, is the requirement of successfully marking the display with intensity I greater than or equal to level I_j, $j = 0, 1, \ldots, 6$ on s, equal to or greater than λ, of the three ping cycles, where $\lambda = 2, 3$, such that the marks line up within plus or minus one resolution interval of the display. The detection criterion may be extended to include marking with different (but definite) minimum intensities on each ping cycle or definite minimum intensities in any order.

B. T is total time for a three-ping history; τ is single ping display time; δt is the display time for a single resolution interval.

C. $P_{\lambda j}(FA)$ is the probability that the detection criterion is met by noise alone, where $\lambda = 2, 3$ and $j = 0, 1 \ldots, 6$.

After a particular detection criterion has been established, the probability of false alarm (that is, the probability that noise alone satisfies the detection criterion) may be computed from the relation[1,2]

*This condition may be relaxed and is treated later.

$$P_{\lambda_j}(FA) = \sum_{k=\lambda}^{3} P'(s = k;\ I \geq I_j),$$

where*

$$P'(s = 3;\ I \geq I_j) = \frac{1}{5}\left(\frac{\tau}{\delta t}\right) [(p_j)(3p_j q_j^2)(3p_j q_j^2)] \text{ and}$$

$$P'(s = 2;\ I \geq I_j) = \frac{1}{5}\left(\frac{\tau}{\delta t}\right) [(p_j^2 q_j^2)\{6q_j^3 + 5q_j^5\}],$$

with $q_j = 1-p_j$ and $(\tau/\delta t) >> 1$.

If different (but definite) minimum intensities are allowed for each ping cycle by the detection criterion, such as $[s \geq \lambda;\ I^1 \geq I_\ell;\ I^2 \geq I_k;\ I^3 \geq I_j]$, then

$$P_{\lambda(jk\ell)}(FA) = \sum_{m=\lambda}^{3} P'[s = m;\ I \geq (j,k,\ell)], \text{ where}$$

$$P'[s = 3;\ (jk\ell)] = \frac{1}{5}(\tau/\delta t)[(p_{1j})(3p_{2k}q_{2k}^2)(3p_{3\ell}q_{3\ell}^2)], \text{and}$$

$$P'[s = 2;\ (jk\ell)] = \frac{1}{5}(\tau/\delta t)[p_{1\ell}(3p_{2k}q_{2k}^2)(q_{3j}^3) + p_{1\ell}q_{2k}^3\ (5p_{3j}q_{3j}^4)$$

$$+ q_{1\ell}^3\ p_{2k}\ (3p_{3j}q_{3j}^2)].$$

If the different minimum intensities can occur in any order, then the P''s will include all combinations of three intensities. For example:

---

*The quantity $(\frac{1}{5})$ is a result of conditions 1 and 4 above.

$P'[s = 3;\ j, k, \ell \text{ any order}]$

$$= \frac{1}{30}(\tau/\delta t)\{(p_{1_j})(3p_{2_k}q_{2_k}{}^2)(3p_{3_\ell}q_{3_\ell}{}^2) + (p_{1_j})(3p_{2_\ell})(3p_{3_k}q_{3_k}{}^2)$$

$$+ (p_{1_k})(3p_{2_j}q_{2_j}{}^2)(3p_{3_\ell}q_{3_\ell}{}^2) + (p_{1_k})(3p_{2_\ell}q_{2_\ell}{}^2)(3p_{3_j}q_{3_j}{}^2)$$

$$+ (p_{1_\ell})(3p_{2_k}q_{2_k}{}^2)(3p_{3_j}q_{3_j}{}^2) + (p_{1_\ell})(3p_{2_j}q_{2_j}{}^2)(3p_{3_k}q_{3_k}{}^2)\} .$$

The false alarm rate, FAR, may be calculated from the following relations:*

$$FAR(\sec^{-1}) = \frac{1}{T}\ln\left(\frac{1}{1 - P(FA)}\right)$$

or

$$FAR(\sec^{-1}) = \frac{P(FA)}{T}, \text{ when } P(FA) << 1 .$$

For a particular detection criterion, the probability of detection, P(D), may be computed with the aid of curves of probability of exceeding a threshold vs threshold, with display input signal-to-noise ratio as a parameter. Since the signal-to-noise ratios for the various ping cycles may well be quite different, it will be assumed that the single ping probability of exceeding threshold, $p_s{}^{\nu j}$, is different for each ping cycle ν. The index "j" is again the intensity level index.

Given the particular detection criterion (such as $[s \geq \lambda;\ I \geq I_j]$) and the three $(p_s{}^{\nu j})$**, the probability of detection $P_{\lambda j}(D)$ may be computed from:

*It is assumed that false alarms are distributed according to a Poisson distribution.

**A procedure will be outlined below for determining the $(p_s{}^{\nu j})$ and the $(p_{\nu j})$.

$$P_{\lambda_j}(D) = \sum_{k=\lambda}^{3} P(S = k;\ I \geq I_j),$$

where

$$P(s = 3;\ I \geq I_j) = [(p_s{}^{1j})(p_s{}^{2j})(p_s{}^{3j})] \text{ and}$$

$$P(s = 2;\ I \geq I_j) = [(p_s{}^{1j})(p_s{}^{2j})(q_s{}^{3j}) + (p_s{}^{1j})(q_s{}^{2j})(p_s{}^{3j})$$

$$+ (q_s{}^{1j})(p_s{}^{2j})(p_s{}^{3j})]$$

If different (but definite) minimum intensities are allowed for each ping cycle by the detection criterion, then the $p_s{}^{\nu j}$ above are changed to

$$p_s{}^{1j} \longrightarrow p_s{}^{1\ell};\ p_s{}^{2j} \longrightarrow p_s{}^{2k};\ p_s{}^{3j} \longrightarrow p_s{}^{3j}.$$

If the different intensities can occur in any order, then the P's above will include all combinations of three intensities. For example:

$$P(s = 3;\ I \geq jk\ell) = \frac{1}{6}\,[(p_s{}^{1\ell})(p_s{}^{2k})(p_s{}^{3j}) + (p_s{}^{1\ell})(p_s{}^{2j})(p_s{}^{3k}) +$$

$$(p_s{}^{1k})(p_s{}^{2\ell})(p_s{}^{3j}) + (p_s{}^{1k})(p_s{}^{2j})(p_s{}^{3\ell}) + (p_s{}^{1j})(P_s{}^{2k})(p_s{}^{3\ell})$$

$$+ (p_s{}^{1j})(p_s{}^{2\ell})(p_s{}^{3k})].$$

The probabilities $p_{\nu j}$ and $p_s{}^{\nu j}$ used above in computing probability of false alarm and probability of detection represent the probability of marking a given resolution interval of the display at an intensity level I_j when the input to the display is noise alone and when it is signal plus noise, respectively. These quantities can be obtained from curves of probability of exceeding a threshold vs threshold with noise alone and with various display input signal-to-noise ratios. Since the marking intensity constitutes an important part of the criteria actually used by sonar operators to call detection, it is desirable that marking intensity be included in a reasonable fashion.

Some data have been advanced which indicate that a detection criterion utilizing six or seven discrete intensity levels may be reasonable even though the actual display marking intensities are proportional to input amplitude[3]. We assume seven discrete levels (the first of which is zero intensity). In order to determine $p_{\nu j}$ and $p_s^{\nu j}$ it is convenient to relate the intensity levels to thresholds. This may be done in the following way. First, the division between the lowest intensity level (zero intensity) and the next lowest is determined by the threshold setting which results in a desirable display clutter density. By specifying display clutter density this threshold, and hence the division between intensity levels 0 and 1, can be determined from a curve of probability of exceeding a threshold vs threshold for noise alone[+]. The division between the highest intensity level and the next highest may be made in a similar, although somewhat more arbitrary, fashion. This intensity level division may be taken tentatively as that threshold which corresponds to a 0.03 probability of exceeding a threshold for the greatest signal-to-noise ratio occurring in the last three ping cycle history.

The relationship between intensity levels and thresholds is completed by dividing the interval of the threshold axis

[+] This curve pertains to the display input. In the case of a display matched to the output of the signal processor, the curve referred to may be that of the processor output.

between the two values established above into five equal segments and labeling the segments intensity levels 1 through 5. The quantity $p_{\nu j}$ may be determined as the probability which corresponds to the intersection of the curve of probability of exceeding a threshold vs threshold for noise alone, and a vertical line passing through the left boundary of the jth intensity level segment. Similarly, the quantity $p_s^{\nu j}$ may be determined as the probability which corresponds to the intersection of the curve of probability of exceeding a threshold vs threshold for the input signal-to-noise ratio of the νth ping cycle, and a vertical line passing through the left boundary of the jth intensity level segment.

4. SIX PING CYCLE HISTORY - ZERO RANGE RATE

4.1 INTRODUCTION

We will consider the case where six ping cycles are displayed after which the earliest three ping cycles are erased and three additional ping cycles are added. The following situations may be distinguished and will be treated separately:

1. The detection criteria may include marking with the required intensity on
   (a) any λ of three ping cycles,
   (b) any λ of four ping cycles,
   (c) any λ of five ping cycles,
   (d) any λ of six ping cycles.
2. The detection criterion may include marking with the required intensity on
   (a) λ consecutive ping cycles of three ping cycles,
   (b) λ consecutive ping cycles of four ping cycles,
   (c) λ consecutive ping cycles of five ping cycles,
   (d) λ consecutive ping cycles of six ping cycles.

We have already considered 1.(a) above which includes also the probabilities required for 2.(a). The required probabilities for 2.(a) will be stated explicitly below, following which cases 1.(b) and 2.(b), 1.(c) and 2.(c), and 1.(d) and 2.(d) will be stated. The procedure for calculating the probability of satisfying the detection criterion on ν ping cycles will be to calculate first the probability of satisfying the detection criterion on $(\nu-1)$ ping cycles and adding to this the probability of satisfying the criterion on the νth ping cycle.

4.2 λ OR MORE CONSECUTIVE MARKS OF THREE PING CYCLES

The probability of any λ or more of three ping cycles has been given previously as:

$$P_{\lambda_j}(FA) = \frac{1}{5}\sum_{k=\lambda}^{3} P'(s=k;\ I\geq I_j),\ \text{where}$$

$$P'(s=3;\ I\geq I_j) = (^{\tau}/\delta t)\ [(p_j)(3p_jq_j^{\,2})(3p_jq_j^{\,2})]$$

and

$$P'(s=2;\ I\geq I_j) = (^{\tau}/\delta t)[(p_j^{\,2}q_j^{\,2})\{6q_j^{\,3} + 5q_j^{\,5}\}],$$

for marks of the same minimum intensity. If the marks are required to be on <u>consecutive</u> ping cycles, then $P'(s=2;\ I\geq I_j)$ above is changed to

$$P_c'(s=2;\ I\geq I_j) = (^{\tau}/\delta t)[(p_j^{\,2}q_j^{\,2})\{6q_j^{\,3}\}],$$

since the last term in $P'(s=2;\ I\geq I_j)$ above represents the probability of marking on the first and third ping cycle and does not satisfy the new requirement. For different but definite minimum intensities $P'[s=2;\ (jk\ell)]$ becomes:

$$P_c'[s=2;(jk\ell)] = (^{\tau}/\delta t)[(p_{2k})(3p_{3\ell}q_{3\ell}^{\,2})xq_{1j}^{3} + (p_{1j})(3p_{2k}q_{2k}^{2})(q_{3\ell}^{3})]\ .$$

After the same fashion the probability of detection may be modified by

$$P(s=2;\ I\geq I_j) \rightarrow P_c(s=2;\ I\geq I_j) = [(p_s^{\,1j})(p_s^{\,2j})(q_s^{\,3j}) + (p_s^{\,2j})(p_s^{\,3j})(q_s^{\,1j})]\ ,$$

or by replacing $p_s^{\,1j}$ with $p_s^{\,1\ell}$ and $p_s^{\,2j}$ with $p_s^{\,2k}$ in the case of different but definite minimum intensities.

4.3 ANY λ OR MORE MARKS ON FOUR PING CYCLES

The probability of satisfying this criterion with noise alone as input is given by

$$P_{\lambda j}(FA) = \frac{1}{7} \sum_{k=\lambda}^{4} P'(s=k \text{ of } 4;\ I \geq I_j) \ .$$

The probabilities $P'(s=k \text{ of } 4;\ I \geq I_j)$ may be computed by adding to $P'(s=k \text{ of } 3;\ I \geq I_j)$ the probability of satisfying (k of 4) on the fourth ping cycle. Thus

- $$P'(s=4 \text{ of } 4;\ I \geq I_j) = (^{\tau}/\delta t)[(p_{1j})(3p_{2j}q_{2j}^2)(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)] \ ,$$

- $$\begin{aligned} P'(s=3 \text{ of } 4;\ I \geq I_j) = {} & P'(s=3 \text{ of } 3;\ I \geq I_j)(q_{4j}^3) \\ & + (^{\tau}/\delta t)[(p_{1j})(3p_{2j}q_{2j}^2)(q_{3j}^3)(5p_{4j}q_{4j}^4) \\ & + (p_{1j})(q_{2j}^3)(5p_{3j}q_{3j}^4)(3p_{4j}q_{4j}^2) \\ & + (q_{1j}^3)(p_{2j})(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)] \ , \end{aligned}$$

and

- $$\begin{aligned} P'(s=2 \text{ of } 4;\ I \geq I_j) = {} & P'(s=2 \text{ of } 3;\ I \geq I_j) \times (q_{4j}^3 \text{ or } q_{4j}^5)^* \\ & + (^{\tau}/\delta t)[(p_{1j})(q_{2j}^3)(q_{3j}^5)(7p_{4j}q_{4j}^6) + (q_{1j}^3)(p_{2j})(q_{3j}^3)(5p_{4j}q_{4j}^4) \\ & + (q_{1j}^5)(q_{2j}^3)(p_{3j})(3p_{4j}q_{4j}^2)] \ . \end{aligned}$$

Similarly, the probability of satisfying the detection criterion with signal plus noise as input is given by

---

*q_{4j}^{3} is used if a hit occurred on the third ping cycle and q_{4j}^{5} is used if a miss occurred.

- $P_{\lambda j}(D)(s \geq \lambda \text{ of } 4; I \geq I_j) = \sum_{k=\lambda}^{4} P(s=k \text{ of } 4; I \geq I_j)$,

where

- $P(s=4; I \geq I_j) = [(p_s^{1j})(p_s^{2j})(p_s^{3j})(p_s^{4j})]$,

- $P(s=3 \text{ of } 4; I \geq I_j) = [P(s=3 \text{ of } 3) \times (q_s^{4j})$

$$+ (p_s^{1j})(p_s^{2j})(q_s^{3j})(p_s^{4j}) + (p_s^{1j})(q_s^{2j})(p_s^{3j})(p_s^{4j})$$

$$+ (q_s^{1j})(p_s^{2j})(p_s^{3j})(p_s^{4j})],$$

and

- $P(s=2 \text{ of } 4; I \geq I_j) = [P(s=2 \text{ of } 3)(q_s^{4j})$

$$+ (p_s^{1j})(q_s^{2j})(q_s^{3j})(p_s^{4j}) + (q_s^{1j})(p_s^{2j})(q_s^{3j})(p_s^{4j})$$

$$+ (q_s^{1j})(q_s^{2j})(p_s^{3j})(p_s^{4j})] .$$

4.4 λ OR MORE CONSECUTIVE MARKS ON FOUR PING CYCLES

The probability of λ or more consecutive marks on four ping cycles may be obtained by replacing each $P'(s=k \text{ of } 4; I \geq I_j)$ above with $P_c'(s=k \text{ of } 4; I \geq I_j)$, where

- $P_c'(s=4 \text{ of } 4) = P'(s=4 \text{ of } 4)$,

- $P_c'(s=3 \text{ of } 4) = P'(s=3 \text{ of } 3) \times q_{4j}^3$

$$+ (\tau/\delta t)[(q_{1j}^3)(3p_{2j}q_{2j}^2)(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)]$$

• $P_c'(s{=}2 \text{ of } 4) = P_c'(s{=}2 \text{ of } 3)\ (q_{4j}^3 \text{ or } q_{4j}^5)$

$$+ (\tau/\delta t)[(q_{1j}^5)(q_{2j}^3)(p_{3j})(3p_{4j}q_{4j}^2)] \ ,$$

and

• $P_c(s{=}4 \text{ of } 4) = P(s{=}4 \text{ of } 4)$,

• $P_c(s{=}3 \text{ of } 4) = [P(s{=}3 \text{ of } 3)x(q_s^{4j}) + (q_s^{1j})(p_s^{2j})(p_s^{3j})(p_s^{4j})]$,

• $P_c(s{=}2 \text{ of } 4) = [P_c(s{=}2 \text{ of } 3)(q_s^{4j}) + (q_s^{1j})(q_s^{2j})(p_s^{3j})(p_s^{4j})]$.

4.5 ANY λ OR MORE MARKS ON FIVE PING CYCLES

The probabilities $P'(s{=}k \text{ of } 5;\ I \ge I_j)$ may be computed by adding to $P'(s{=}k \text{ of } 4;\ I \ge I_j)$ the probability of satisfying (k of 5) on the fifth ping cycle. Thus

• $P'(s{=}5 \text{ of } 5;\ I \ge I_j) = (\tau/\delta t)[(p_{1j})(3p_{2j}q_{2j}^2)(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)]$,

• $P'(s{=}4 \text{ of } 5;\ I \ge I_j) = P'(s{=}4 \text{ of } 4;\ I \ge I_j)(q_{5j}^3)$

$$+ (\tau/\delta t)[(q_{1j}^3)(p_{2j})(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)$$

$$+ (p_{1j})(q_{2j}^3)(5p_{3j}q_{3j}^4)(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)$$

$$+ (p_{1j})(3p_{2j}q_{2j}^2)(q_{3j}^3)(5p_{4j}q_{4j}^4)(3p_{5j}q_{5j}^2)$$

$$+ (p_{1j})(3p_{2j}q_{2j}^2)(3p_{3j}q_{3j}^2)(q_{4j}^3)(5p_{5j}q_{5j}^4)] \ ,$$

- $P'(s=3 \text{ of } 5;\ I \geq I_j) = P'(s=3 \text{ of } 4;\ I \geq I_j) \times (q_{5j}^3 \text{ or } q_{5j}^5)$

$$+ (^{\tau}/\delta t)[(q_{1j}^3)(p_{2j})(3p_{3j}q_{3j}^2)(q_{4j}^3)(5p_{5j}q_{5j}^4)$$

$$+ (q_{1j}^3)(p_{2j})(q_{3j}^3)(5p_{4j}q_{4j}^4)(3p_{5j}q_{5j}^2)$$

$$+ (q_{1j}^5)(q_{2j}^3)(p_{3j})(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)$$

$$+ (p_{1j})(3p_{2j}q_{2j}^2)(q_{3j}^3)(q_{4j}^5)(7p_{5j}q_{5j}^6)$$

$$+ (p_{1j})(q_{2j}^3)(5p_{3j}q_{3j}^4)(q_{4j}^3)(5p_{5j}q_{5j}^4)$$

$$+ (p_{1j})(q_{2j}^3)(q_{3j}^5)(7p_{4j}q_{4j}^6)(3p_{5j}q_{5j}^2)] ,$$

and

- $P'(s=2 \text{ of } 5;\ I \geq I_j) = P'(s=2 \text{ of } 4;\ I \geq I_j)(q_{5j}^3,\ q_{5j}^5 \text{ or } q_{5j}^7)$

$$+ (^{\tau}/\delta t)[(p_{1j})(q_{2j}^3)(q_{3j}^5)(q_{4j}^7)(9p_{5j}q_{5j}^8)$$

$$+ (q_{1j}^3)(p_{2j})(q_{3j}^3)(q_{4j}^5)(7p_{5j}q_{5j}^6)$$

$$+ (q_{1j}^5)(q_{2j}^3)(p_{3j})(q_{4j}^3)(5p_{5j}q_{5j}^4)$$

$$+ (q_{1j}^7)(q_{2j}^5)(q_{3j}^3)(p_{4j})(3p_{5j}q_{5j}^2)] .$$

Similarly, the probabilities $P(s=k \text{ of } 5;\ I \geq I_j)$ may be computed by adding to $P(s=k \text{ of } 4;\ I \geq I_j)$ the probability of satisfying (k of 5) on the fifth ping cycle. Thus

- $P(s=5 \text{ of } 5;\ I \geq I_j) = \prod_{\nu=1}^{5} P_s^{\nu j}$,

- $P(s=4 \text{ of } 5;\ I \geq I_j) = P(s=4 \text{ of } 4)(q_s^{5j})$

$$+ [(p_s^{5j})\{(p_s^{4j})(p_s^{3j})(p_s^{2j})(q_s^{1j}) + (p_s^{4j})(p_s^{3j})(q_s^{2j})(p_s^{1j})$$
$$+ (p_s^{4j})(q_s^{3j})(p_s^{2j})(p_s^{1j}) + (q_s^{4j})(p_s^{3j})(p_s^{2j})(p_s^{1j})\}] \ ,$$

- $P(s=3 \text{ of } 5;\ I \geq I_j) = P(s=3 \text{ of } 4)(q_s^{5j})$

$$+ [(p_s^{5j})\{(p_s^{4j})(p_s^{3j})(q_s^{2j})(q_s^{1j}) + (p_s^{4j})(p_s^{2j})(q_s^{3j})(q_s^{1j})$$
$$+ (q_s^{4j})(p_s^{3j})(p_s^{2j})(q_s^{1j}) + (p_s^{4j})(q_s^{3j})(q_s^{2j})(p_s^{1j})$$
$$+ (q_s^{4j})(p_s^{3j})(q_s^{2j})(p_s^{1j}) + (q_s^{4j})(q_s^{3j})(p_s^{2j})(p_s^{1j})\}] ,$$

and

- $P(s=2 \text{ of } 5;\ I \geq I_j) = P(s=2 \text{ of } 4)(q_s^{5j})$

$$+ [(p_s^{5j})\{(p_s^{4j})(q_s^{3j})(q_s^{2j})(q_s^{1j}) + (q_s^{4j})(p_s^{3j})(q_s^{2j})(q_s^{1j})$$
$$+ (q^{4j})(q^{3j})(p^{2j})(q^{1j}) + (q^{4j})(q^{3j})(q^{2j})(p^{1j})\}] \ .$$

4.6 λ OR MORE CONSECUTIVE MARKS ON FIVE PING CYCLES

The probability of λ or more consecutive marks on five ping cycles may be obtained by replacing each $P'(s=k \text{ of } 5;\ I \geq I_j)$ with $P'_c(s=k \text{ of } 5;\ I \geq I_j)$ and each $P(s=k \text{ of } 5;\ I \geq I_j$ with $P_c(s=k \text{ of } 5;\ I \geq I_j)$, where

- $P'_c(s=5 \text{ of } 5) = P'(s=5 \text{ of } 5)$,

- $P'_c(s=4 \text{ of } 5) = P'(s=4 \text{ of } 4)(q_{5j}^3)$

$$+ (\tau/\delta t)[(q_{1j}^3)(p_{2j})(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)] \ ,$$

- $P_c'(s=3 \text{ of } 5) = P_c'(s=3 \text{ of } 4)(q_{5j}^3 \text{ or } q_{5j}^5)$

$$+ (^{\tau}/\delta t)[(q_{1j}^5)(q_{2j}^3)(p_{3j})(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)] \ ,$$

- $P_c'(s=2 \text{ of } 5) = P_c'(s=2 \text{ of } 4)(q_{5j}^3,\ q_{5j}^5 \text{ or } q_{5j}^7)$

$$+ (^{\tau}/\delta t)[(q_{1j}^7)(q_{2j}^5)(q_{3j}^3)(p_{4j})(3p_{5j}q_{5j}^2)] \ .$$

Also,

- $P_c(s=5 \text{ of } 5) = P(s=5 \text{ of } 5)$,

- $P_c(s=4 \text{ of } 5) = P(s=4 \text{ of } 4)(q_s^{5j}) + [(q_s^{1j})(p_s^{2j})(p_s^{3j})(p_s^{4j})(p_s^{5j})]$,

- $P_c(s=3 \text{ of } 5) = P_c(s=3 \text{ of } 4)(q_s^{5j}) + [(q_s^{1j})(q_s^{2j})(p_s^{3j})(p_s^{4j})(p_s^{5j})]$,

and

- $P_c(s=2 \text{ of } 5) = P_c(s=2 \text{ of } 4)(q_s^{5j}) + [(q_s^{1j})(q_s^{2j})(q_s^{3j})(p_s^{4j})(p_s^{5j})]$.

4.7 ANY λ OR MORE MARKS ON SIX PING CYCLES

As before, the probabilities $P'(s=k \text{ of } 6;\ I \geq I_j)$ may be computed by adding to $P'(s=k \text{ of } 5;\ I \geq I_j)$ the probability of satisfying (k of 6) on the sixth ping cycle. Thus

- $P'(s=6 \text{ of } 6;\ I \geq I_j) = (^{\tau}/\delta t)[(p_{1j})(3p_{2j}q_{2j}^2)(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)$

$$\cdot\ (3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)] ,$$

- $P'(s=5 \text{ of } 6; I \geq I_j) = P'(s=5 \text{ of } 5; I \geq I_j)(q_{6j}^3) +$

$$(\tau/\delta t)[(p_{1j})(3p_{2j}q_{2j}^2)(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)(q_{5j}^3)(5p_{6j}q_{6j}^4)$$
$$+ (p_{1j})(3p_{2j}q_{2j}^2)(3p_{3j}q_{3j}^2)(q_{4j}^3)(5p_{5j}q_{5j}^4)(3p_{6j}q_{6j}^2)$$
$$+ (p_{1j})(3p_{2j}q_{2j}^2)(q_{3j}^3)(5p_{4j}q_{4j}^4)(3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)$$
$$+ (p_{1j})(q_{2j}^3)(5p_{3j}q_{3j}^4)(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)$$
$$+ (q_{1j}^3)(p_{2j})(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)] ,$$

- $P'(s=4 \text{ of } 6; I \geq I_j) = P'(s=4 \text{ of } 5; I \geq I_j)(q_{6j}^3)$ or (q_{6j}^5)

$$+ (\tau/\delta t)[(p_{1j})(3p_{2j}q_{2j}^2)(3p_{3j}q_{3j}^2)(q_{4j}^3)(q_{5j}^5)(7p_{6j}q_{6j}^6)$$
$$+ (p_{1j})(3p_{2j}q_{2j}^2)(q_{3j}^3)(5p_{4j}q_{4j}^4)(q_{5j}^3)(5p_{6j}q_{6j}^4)$$
$$+ (p_{1j}(q_{2j}^3)(5p_{3j}q_{3j}^4)(3p_{4j}q_{4j}^2)(q_{5j}^3)(5p_{6j}q_{6j}^4)$$
$$+ (q_{1j}^3)(p_{2j})(3p_{3j}q_{3j}^2)(3p_{4j}q_{4j}^2)(q_{5j}^3)(5p_{6j}q_{6j}^4)$$
$$+ (p_{1j})(3p_{2j}q_{2j}^2)(q_{3j}^3)(q_{4j}^5)(7p_{5j}q_{5j}^6)(3p_{6j}q_{6j}^2)$$
$$+ (p_{1j})(q_{2j}^3)(5p_{3j}q_{3j}^4)(q_{4j}^3)(5p_{5j}q_{5j}^4)(3p_{6j}q_{6j}^2)$$
$$+ (q_{1j}^3)(p_{2j})(3p_{3j}q_{3j}^2)(q_{4j}^3)(5p_{5j}q_{5j}^4)(3p_{6j}q_{6j}^2)$$
$$+ (p_{1j})(q_{2j}^3)(q_{3j}^5)(7p_{4j}q_{4j}^6)(3p_{6j}q_{5j}^2)(3p_{6j}q_{6j}^2)$$
$$+ (q_{1j}^3)(p_{2j})(q_{3j}^3)(5p_{4j}q_{4j}^4)(3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)$$
$$+ (q_{1j}^5)(q_{2j}^3)(p_{3j})(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)]$$

- $P'(s=3 \text{ of } 6;\ I \geq I_j) = P'(s=3 \text{ of } 5;\ I \geq I_j)(q_{6j}^3,\ q_{6j}^5 \text{ or } q_{6j}^7)$

$$
\begin{aligned}
(\tau/\delta t)[\,&(p_{1j})(3p_{2j}q_{2j}^2)(q_{3j}^3)(q_{4j}^5)(q_{5j}^7)(9p_{6j}q_{6j}^8)\\
&+ (p_{1j})(q_{2j}^3)(5p_{3j}q_{3j}^4)(q_{4j}^3)(q_{5j}^5)(7p_{6j}q_{6j}^6)\\
&+ (q_{1j}^3)(p_{2j})(3p_{3j}q_{3j}^2)(q_{4j}^3)(q_{5j}^5)(7p_{6j}q_{6j}^6)\\
&+ (p_{1j})(q_{2j}^3)(q_{3j}^5)(7p_{4j}q_{4j}^6)(q_{5j}^3)(5p_{6j}q_{6j}^4)\\
&+ (q_{1j}^3)(p_{2j})(q_{3j}^3)(5p_{4j}q_{4j}^4)(q_{5j}^3)(5p_{6j}q_{6j}^4)\\
&+ (q_{1j}^5)(q_{2j}^3)(p_{3j})(3p_{4j}q_{4j}^2)(q_{5j}^3)(5p_{6j}q_{6j}^4)\\
&+ (p_{1j}(q_{2j}^3)(q_{3j}^5)(q_{4j}^7)(9p_{5j}q_{5j}^8)(3p_{6j}q_{6j}^2)\\
&+ (q_{1j}^3)(p_{2j})(q_{3j}^3)(q_{4j}^5)(7p_{5j}q_{5j}^6)(3p_{6j}q_{6j}^2)\\
&+ (q_{1j}^5)(q_{2j}^3)(p_{3j})(q_{4j}^3)(5p_{5j}q_{5j}^4)(3p_{6j}q_{6j}^2)\\
&+ (q_{1j}^7)(q_{2j}^5)(q_{3j}^3)(p_{4j})(3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)]\ ,
\end{aligned}
$$

- $P'(s=2 \text{ of } 6;\ I \geq I_j) = P'(s=2 \text{ of } 5)(q_{6j}^3,\ q_{6j}^5,\ q_{6j}^7 \text{ or } q_{6j}^9)$

$$
\begin{aligned}
(\tau/\delta t)[\,&(p_{1j}(q_{2j}^3)(q_{3j}^5)(q_{4j}^7)(q_{5j}^9)(11p_{6j}q_{6j}^{10})\\
&+ (q_{1j}^3)(p_{2j})(q_{3j}^3)(q_{4j}^5)(q_{5j}^7)(9p_{6j}q_{6j}^8)\\
&+ (q_{1j}^5)(q_{2j}^3)(p_{3j})(q_{4j}^3)(q_{5j}^5)(7p_{6j}q_{6j}^6)\\
&+ (q_{1j}^7)(q_{2j}^5)(q_{3j}^3)(p_{4j})(q_{5j}^3)(5p_{6j}q_{6j}^4)\\
&+ (q_{1j}^9)(q_{2j}^7)(q_{3j}^5)(q_{4j}^3)(p_{5j})(3p_{6j}q_{6j}^2)]\ .
\end{aligned}
$$

Also,

- $P(s=6 \text{ of } 6;\ I \geq I_j) = \prod_{\nu=1}^{6} p_s^{\nu j}$,

- $P(s=5 \text{ of } 6;\ I \geq I_j) = P(s=5 \text{ of } 5)(q_s^{6j}) + (p_s^{6j})[(q_s^{1j})(p_s^{2j})(p_s^{3j})(p_s^{4j})(p_s^{5j})$

  $+ (p_s^{1j})(q_s^{2j})(p_s^{3j})(p_s^{4j})(p_s^{5j}) + (p_s^{1j})(p_s^{2j})(q_s^{3j})(p_s^{4j})(p_s^{5j})$

  $+ (p_s^{1j})(p_s^{2j})(p_s^{3j})(q_s^{4j})(p_s^{5j}) + (p_s^{1j})(p_s^{2j})(p_s^{3j})(p_s^{4j})(q_s^{5j})]$,

- $P(s=4 \text{ of } 6;\ I \geq I_j) = P(s=4 \text{ of } 5)(q_s^{6j}) + (p_s^{6j})\Big[(q_s^{5j})\{(p_s^{4j})(p_s^{3j})(p_s^{2j})(q_s^{1j})$

  $+ (p_s^{4j})(p_s^{3j})(q_s^{2j})(p_s^{1j}) + (p_s^{4j})(q_s^{3j})(p_s^{2j})(p_s^{1j}) + (q_s^{4j})(p_s^{3j})(p_s^{2j})(p_s^{1j})\}$

  $+ (p_s^{5j})\{(q_s^{4j})(p_s^{3j})(p_s^{2j})(q_s^{1j}) + (q_s^{4j})(p_s^{3j})(q_s^{2j})(p_s^{1j})$

  $+ (q_s^{4j})(q_s^{3j})(p_s^{2j})(p_s^{1j}) + (p_s^{4j})[(q_s^{3j})(p_s^{2j})(q_s^{1j})$

  $+ (q_s^{3j})(q_s^{2j})(p_s^{1j}) + (p_s^{3j})(q_s^{2j})(q_s^{1j})]\}\Big]$,

- $P(s=3 \text{ of } 6;\ I \geq I_j) = P(s=3 \text{ of } 5)(q_s^{6j}) + (p_s^{6j})[(q_s^{1j})\{(q_s^{2j})(q_s^{3j})(p_s^{4j})(p_s^{5j})$

  $+ (q_s^{2j})(p_s^{3j})(q_s^{4j})(p_s^{5j}) + (q_s^{2j})(p_s^{3j})(p_s^{4j})(q_s^{5j}) + (p_s^{2j})(q_s^{3j})(q_s^{4j})(p_s^{5j})$

  $+ (p_s^{2j})(q_s^{3j})(p_s^{4j})(q_s^{5j}) + (p_s^{2j})(p_s^{3j})(q_s^{4j})(q_s^{5j})\}$

  $+ (p_s^{1j})\{(q_s^{2j})(q_s^{3j})(q_s^{4j})(p_s^{5j}) + (q_s^{2j})(q_s^{3j})(p_s^{4j})(q_s^{5j})$

  $+ (q_s^{2j})(p_s^{3j})(q_s^{4j})(q_s^{5j}) + (p_s^{2j})(q_s^{3j})(q_s^{4j})(q_s^{5j})\}]$,

• $P(s=2 \text{ of } 6;\ I \geq I_j) = P(s=2 \text{ of } 5)(q_s^{6j}) + (p_s^{6j})[(p_s^{5j})(q_s^{4j})(q_s^{3j})(q_s^{2j})(q_s^{1j})$

$+ (q_s^{5j})(p_s^{4j})(q_s^{3j})(q_s^{2j})(q_s^{1j}) + (q_s^{5j})(q_s^{4j})(p_s^{3j})(q_s^{2j})(q_s^{1j})$

$+ (q_s^{5j})(q_s^{4j})(q_s^{3j})(p_s^{2j})(q_s^{1j}) + (q_s^{5j})(q_s^{4j})(q_s^{3j})(q_s^{2j})(p_s^{1j})]$.

4.8 λ OR MORE CONSECUTIVE MARKS ON SIX PING CYCLES

The probability of λ or more consecutive marks on six ping cycles may be obtained by replacing each $P'(s=k \text{ of } 6;\ I \geq I_j)$ with $P_c'(s=k \text{ of } 6;\ I \geq I_j)$ and each $P(s=k \text{ of } 6;\ I \geq I_j)$ with $P_c(s=k \text{ of } 6;\ I \geq I_j)$, where

• $P_c'(s=6 \text{ of } 6) = P'(s=6 \text{ of } 6)$,

• $P_c'(s=5 \text{ of } 6) = P'(s=5 \text{ of } 5)(q_{6j}^3) + (^\tau/\delta t)[(q_{1j}^3)(p_{2j})(3p_{3j}q_{3j}^2) \cdot$

$(3p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)]$,

• $P_c'(s=4 \text{ of } 6) = P_c'(s=4 \text{ of } 5)[(q_{6j}^3) \text{ or } (q_{6j}^5)] + (^\tau/\delta t)[(q_{1j}^5)(q_{2j}^3) \cdot$

$(p_{3j})(2p_{4j}q_{4j}^2)(3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)]$,

• $P_c'(s=3 \text{ of } 6) = P_c'(s=3 \text{ of } 5)[q_{6j}^7,\ q_{6j}^5 \text{ or } q_{6j}^3] + (^\tau/\delta t)[(q_{1j}^7)(q_{2j}^5) \cdot$

$(q_{3j}^3)(p_{4j})(3p_{5j}q_{5j}^2)(3p_{6j}q_{6j}^2)]$,

• $P_c'(s=2 \text{ of } 6) = P_c'(s=2 \text{ of } 5)[q_{6j}^9,\ q_{6j}^7,\ q_{6j}^5 \text{ or } q_{6j}^3] + (^\tau/\delta t)[(q_{1j}^9) \cdot$

$(q_{2j}^7)(q_{3j}^5)(q_{4j}^3)(p_{5j})(3p_{6j}q_{6j}^2)]$;

also

- $P_c(s=6 \text{ of } 6) = P(s=6 \text{ of } 6)$,

- $P_c(s=5 \text{ of } 6) = P(s=5 \text{ of } 5)(q_s^{6j}) + [(q_s^{1j})(p_s^{2j})(p_s^{3j})(p_s^{4j})(p_s^{5j})(p_s^{6j})]$,

- $P_c(s=4 \text{ of } 6) = P_c(s=4 \text{ of } 5)(q_s^{6j}) + [(q_s^{1j})(q_s^{2j})(p_s^{3j})(p_s^{4j})(p_s^{5j})(p_s^{6j})]$,

- $P_c(s=3 \text{ of } 6) = P_c(s=3 \text{ of } 5)(q_s^{6j}) + [(q_s^{1j})(q_s^{2j})(q_s^{3j})(p_s^{4j})(p_s^{5j})(p_s^{6j})]$,

and

- $P_c(s=2 \text{ of } 6) = P_c(s=2 \text{ of } 5)(q_s^{6j}) + [(q_s^{1j})(q_s^{2j})(q_s^{3j})(q_s^{4j})(p_s^{5j})(p_s^{6j})]$.

4.9 INTENSITY OF MARKS ALLOWED TO VARY FROM PING TO PING

In the event each mark is allowed a different but definite minimum intensity, the probabilities $p_{\nu j}$, $q_{\nu j}$, $p_s^{\nu j}$ and $q_s^{\nu j}$ above are each replaced with probabilities which correspond to their respective minimum intensities in the manner indicated previously for three ping cycles. If a definite set of minimum intensities is considered in which the intensity of marks can occur in any order, then all combinations of intensity marks within the set must be considered. This also has been indicated for a three ping cycle history.

Limits must be placed upon the lowest intensity which may be included in any observer criteria in order to remain consistent with an assumption implicit in the above probability statements. This assumption is that the probability of more than one mark occurring in any ping cycle leading to more than one possible false alarm path at a given range can be ignored.

4.10 ADDITIONAL CONSIDERATION

After the initial six ping cycles there are always at least three ping cycles being displayed. Following the sixth ping cycle the earliest three are erased and the latest three retained. To these three are added a new fourth, fifth, and sixth cycle. This process is repeated after each sixth ping cycle is displayed. Since the latest three ping cycles are retained, the probabilities for satisfying the three ping cycle history criterion should be calculated following the sixth ping cycle as well as those for satisfying a six ping cycle criterion. The three ping cycle probabilities are calculated on the basis of the last three ping cycles.

Statements of false alarm probability are given for a single beam. The false alarm probability or false alarm rate for the display would require consideration of the number of beams being displayed. This could increase the single beam false alarm probability by an order of magnitude or more.

5. NON-ZERO RANGE RATES

The detection criteria and, specifically, the vacillation of marks from ping to ping have been discussed from the standpoint of zero range rate targets. To be of more than academic interest non-zero range rate detection criteria should also be established. A brief study has been reported[4] of the effect of target range rate on observers ability to detect signals. Two significant points were reported: First, observer performance was consistently poorer for non-zero range rate signals than for zero range rate signals. Second, observer performance with zero range rate signals was poorer when the possibility of non-zero range rate signals was allowed than when only zero range rate signals were allowed.

When detection criteria are established in an effort to match observer curves, care must be used in choosing criteria which correspond to realistic operational requirements. It is doubtful that one would exclude the possibility of non-zero range rate targets. Limits could be established for range rates which are reasonable to include.

The question of what detection criteria to choose when considering non-zero range rate targets should be answered in terms of operator alertness condition (e.g., bearing alerted, range alerted, range and bearing alerted, or unalerted). Without sufficient experimental data on which to base choices of detection criteria, one can only establish criteria upon existing information with the expectation that these will likely change as experimental data become more complete.

Consider operation in a bottom bounce mode, in 3,000 fathoms of water, and with a 20 degree depression angle. The above leads to a ping interval of about 48 seconds and a display time of about 13 seconds. Non-zero range rate targets in the illuminated zone (approximately 25 to 35 kyd) will cause corresponding echo marks on consecutive ping cycles to be displaced about 1 resolution interval per knot of range rate. If we allow possible targets a range rate capability of +n to -m knots, marks on consecutive ping cycles which are misaligned by +n to -m resolution intervals may represent a possible target. The possible misalignment on non-consecutive ping cycles increases with the number of ping cycles in the following manner:

(a) 2n to -2m from first to third ping cycle,
(b) 3n to -3m from first to fourth ping cycle,
(c) 4n to -4m from first to fifth ping cycle, and
(d) 5n to -5m from first to sixth ping cycle.

For more than two marks, the possible positions of the third and higher numbers of marks may be determined by restrictions on target rate of change of range rate. Since two marks determine a possible range rate, the third and higher marks would be expected to appear near the intersection of the corresponding ping cycle and a line joining the first two marks if the range rate remained constant. If targets are allowed the capability of changing the range rate, deviations from the expected alignment may occur within the limitations of allowed rate of change of range rate capabilities of possible targets.

Assuming a target rate of change of range rate capability sufficient to cause deviation from an indicated range rate of ±k resolution intervals per ping cycle and a target range rate capability sufficient to cause misalignment of marks by +n to -m resolution intervals per ping cycle, the probability of false alarm can be computed for the criteria discussed earlier. Before giving a few examples, it is convenient at this point to discuss an assumption implicit in the previous computations for zero range rate targets. This is the assumption of only one mark (of the required intensity) falling in the interval determined by the allowance for possible misalignment of marks. For non-zero range rates, the interval becomes larger and the assumption of only one mark may be invalid for even moderate marking densities at the lower intensities. We will thus restrict our detection criteria to include only intensities such that the probability of two marks is less than or equal to one-tenth the probability of one mark in the allowed interval. (e.g., form the ratio

$$P(s = 2) / P(s = 1) = [(\gamma-1)m + (\gamma-1)n] p_{2j} q_{2j}^{-1} / 2,$$

where γ is the number of ping cycles, and consider only marks of intensity $I \geq I_j$ such that $P(s = 2) / P(s = 1) \leq 0.1$.)

With the above restrictions placed on range rate, rate of change of range rate, and minimum intensities we will proceed with the computation of false alarm probabilities for various detection criteria.

5.1 ANY λ OR MORE OF THREE PING CYCLES

The probability of any λ or more of three ping cycles may be obtained from the following relation:

$$P_{\lambda j}(FA) = \frac{1}{(2n+2m+1)} \sum_{\ell=\lambda}^{3} P'(s=\ell;\ I \geq I_j), \text{ where}$$

$$P'(s=3;\ I \geq I_j) = (\tau/\delta t)\ [(p_{1_j})\ \{(n+m+1)\ p_{2_j} q_{2_j}^{\ n+m}\}\ \{(2k+1)\ p_{3_j} q_{3_j}^{\ 2k}\}]$$

and

$$P'(s=2;\ I \geq I_j) = (\tau/\delta t)\ [(p_{1_j})\ \{(n+m+1)\ p_{2j} q_{2j}^{\ n+m}\}\ (q_{3j}^{\ 2k+1})$$

$$+ (p_{1j})\ (q_{2j}^{\ n+m+1})\ \{(2n+2m+1)\ p_{3j} q_{3j}^{\ 2n+2m}\} + (q_{1j}^{\ n+m+1})\ \{p_{2j}\}$$

$$\{(n+m+1)\ p_{3j} q_{3j}^{\ n+m}\}].$$

The obvious reduction is made for λ or more <u>consecutive</u> marks on three ping cycles.

5.2 ANY λ OR MORE OF SIX PING CYCLES

Similarly to the case above, the probability of false alarm may be obtained from the relation

$$P_{\lambda j}(FA) = \frac{1}{(5n+5m+1)} \sum_{\ell=\lambda}^{6} P'(s=\ell;\ I \geq I_j),\ \text{where}$$

- $$P'(s=6;\ I\geq I_j) = (\tau/\delta t)[(p_{1j})\{(n+m+1)p_{2j}q_{2j}^{\ n+m}\}\{2k+1)p_{3j}q_{3j}^{\ 2k}\}$$
$$\{(2k+1)p_{4j}q_{4j}^{\ 2k}\}\{(2k+1)p_{5j}q_{5j}^{\ 2k}\}\{(2k+1)p_{6j}q_{6j}^{\ 2k}\}],$$

- $$P'(s=5;\ I\geq I_j) = (\tau/\delta t)[(p_{1j})\{(n+m+1)p_{2j}q_{2j}^{\ n+m}\}\{(2k+1)p_{3j}q_{3j}^{\ 2k}\}$$
$$\times\ \{(2k+1)p_{4j}q_{4j}^{\ 2k}\}\{2k+1)p_{5j}q_{5j}^{\ 2k}\}(q_{6j}^{\ 2k+1}) + (p_{1j})\{(n+m+1)p_{2j}q_{2j}^{\ n+m}\}$$
$$\times\ \{(2k+1)p_{3j}q_{3j}^{\ 2k}\}\ \{(2k+1)p_{4j}q_{4j}^{\ 2k}\}\ (q_{5j}^{\ 2k+1})\ \{(4k+1)p_{6j}q_{6j}^{\ 4k}\}$$
$$+\ (p_{1j})\{(n+m+1)p_{2j}q_{2j}^{\ n+m}\}\{2k+1)p_{3j}q_{3j}^{\ 2k}\}(q_{4j}^{\ 2k+1})\{(4k+1)p_{5j}q_{5j}^{\ 4k}\}$$
$$\times\ \{2k+1)p_{6j}q_{6j}^{\ 2k}\}$$
$$+(p_{1j})\{(n+m+1)p_{2j}q_{2j}^{\ n+m}\}(q_{3j}^{\ 2k+1})\{(4k+1)p_{4j}q_{4j}^{\ 4k}\}\{(2k+1)p_{5j}q_{5j}^{\ 2k}$$
$$\times\ \{(2k+1)p_{6j}q_{6j}^{\ 2k}\}$$

$$+ (p_{1j})(q_{2j}^{n+m+1})\{(2n+2m+1)p_{3j}q_{3j}^{2n+2m}\}\{(2k+1)p_{4j}q_{4j}^{2k}\}\{(2k+1)$$

$$\times p_{5j}q_{5j}^{2k}\}\{(2k+1)p_{6j}q_{6j}^{2k}\}$$

$$+(q_{1j}^{n+m+1})(p_{2j})\{(n+m+1)p_{3j}q_{3j}^{n+m}\}\{2k+1)p_{4j}q_{4j}^{2k}\}\{2k+1)p_{5j}q_{5j}^{2k}\}$$

$$\{(2k+1)p_{6j}q_{6j}^{2k}\}],$$

• $P'(s=4;\ I \geq I_j) = (\tau/\delta t)[(p_{1j})\{(n+m+1)p_{2j}q_{2j}^{n+m}\}\{(2k+1)p_{3j}q_{3j}^{2k}\}$

$$\times\{(2k+1)p_{4j}q_{4j}^{2k}\}(q_{5j}^{2k+1})(q_{6j}^{4k+1}) + (p_{1j})\{(n+m+1)p_{2j}q_{2j}^{n+m}\}$$

$$\{(2k+1)p_{3j}q_{3j}^{2k}\}(q_{4j}^{2k+1})$$

$$\times\ \{(4k+1)p_{5j}q_{5j}^{4k}\}(q_{6j}^{2k+1}) + (p_{1j})(q_{2j}^{n+m+1})\{(2n+2m+1)p_{3j}q_{3j}^{2n+2m}\}$$

$$\times\ \{(2k+1)p_{4j}q_{4j}^{2k}\}\{(2k+1)p_{5j}q_{5j}^{2k}\}(q_{6j}^{2k+1})+(q_{1j}^{n+m+1})(p_{2j})\{(n+m+1)$$

$$\times\ p_{3j}q_{3j}^{n+m}\}$$

$$x\ \{(2k+1)p_{4j}q_{4j}^{2k}\}\{(2k+1)p_{5j}q_{5j}^{2k}\}\{(q_{6j}^{2k+1})+(p_{1j})2\{(n+m+1)p_{2j}q_{2j}^{n+m}\}$$

$$\{(2k+1)p_{3j}q_{3j}^{2k}\}$$

$$x\ (q_{4j}^{2k+1})(q_{5j}^{4k+1})\{(6k+1)p_{6j}q_{6j}^{6k}\} + (p_{1j})\{(n+m+1)p_{2j}q_{2j}^{n+m}\}$$

$$x\ (q_{3j}^{2k+1})\{(4k+1)p_{4j}q_{4j}^{4k}\}\{(2k+1)p_{5j}q_{5j}^{2k}\}(q_{6j}^{2k+1})+(p_{1j})$$

$$x\ \{(n+m+1)p_{2j}q_{2j}^{n+m}\}$$

$$x\ (q_{3j}^{2k+1})\{(4k+1)p_{4j}q_{4j}^{4k}\}(q_{5j}^{2k+1})\{4k+1)p_{6j}q_{6j}^{4k}\}+(p_{1j})(q_{2j}^{n+m+1})$$

$$\{(2n+2m+1)p_{3j}q_{3j}^{2n+2m}\}$$

$$x\ \{(2k+1)p_{4j}q_{4j}^{2k}\}(q_{5j}^{2k+1})\{(4k+1)p_{6j}q_{6j}^{4k}\}+(q_{1j}^{n+m+1})(p_{2j})\{(n+m+1)$$

$$p_{3j}q_{3j}^{n+m}\ \}$$

$$x\ \{(2k+1)p_{4j}q_{4j}^{2k}\}(q_{5j}^{2k+1})\{4k+1)p_{6j}q_{6j}^{4k}\}+(p_{1j})\{(n+m+1)p_{2j}q_{2j}^{n+m}\}$$

$$(q_{3j}^{2k+1})(q_{4j}^{4k+1})$$

$$\times \{(6k+1)p_{5j}q_{5j}{}^{6k}\}\{(2k+1)p_{6j}q_{6j}{}^{2k}\}+(p_{1j})(q_{2j}{}^{n+m+1})\{(2n+2m+1)$$

$$\times\ p_{3j}q_{3j}{}^{2n+2m}\}(q_{4j}{}^{2k+1})$$

$$\times \{(4k+1)p_{5j}q_{5j}{}^{4k}\}\}\{(2k+1)p_{6j}q_{6j}{}^{2k}\}$$

$$+ (q_{1j}{}^{n+m+1})(p_{2j})\{(n+m+1)p_{3j}q_{3j}{}^{n+m}\}(q_{4j}{}^{2k+1})\{(4k+1)p_{5j}q_{5j}{}^{4k}\}$$

$$\times \{(2k+1)p_{6j}q_{6j}{}^{2k}\}$$

$$+ (p_{1j})(q_{2j}{}^{n+m+1})(q_{3j}{}^{2n+2m+1})\{(3n+3m+1)p_{4j}q_{4j}{}^{3n+3m}\}\{(2k+1)p_{5j}q_{5j}{}^{2k}\}$$

$$\times \{(2k+1)p_{6j}q_{6j}{}^{2k}\}$$

$$+ (q_{1_j}{}^{n+m+1})(p_{2_j})(q_{3_j}{}^{n+m+1})\left\{(2n+2m+1)p_{4_j}q_{4_j}{}^{2n+2m}\right\}\left\{(2k+1)p_{5_j}q_{5_j}{}^{2k}\right\}$$

$$\times \left\{(2k+1)p_{6_j}q_{6_j}{}^{2k}\right\}$$

$$+ (q_{1_j}{}^{2n+2m+1})(q_{2_j}{}^{n+m+1})(p_{3_j})\left\{(n+m+1)p_{4_j}q_{4_j}{}^{n+m}\right\}\left\{(2k+1)p_{5_j}q_{5_j}{}^{2k}\right\}$$

$$\times \left\{(2k+1)p_{6_j}q_{6_j}{}^{2k}\right\}\Big],$$

- $P'(s=3;I\geq I_j) =$

$$(\tau/\delta t)\Big[(p_{1_j})\left\{(n+m+1)p_{2_j}q_{2_j}{}^{n+m}\right\}\left\{(2k+1)p_{3_j}q_{3_j}{}^{2k}\right\}(q_{4_j}{}^{2k+1})(q_{5_j}{}^{4k+1})$$

$$\times\ (q_{6_j}{}^{6k+1})$$

$$+ (p_{1_j})\left\{(n+m+1)p_{2_j}q_{2_j}{}^{n+m}\right\}(q_{3_j}{}^{2k+1})\left\{(4k+1)p_{4_j}q_{4_j}{}^{4k}\right\}(q_{5_j}{}^{2k+1})(q_{6_j}{}^{4k+1})$$

$$+ (p_{1_j})(q_{2_j}^{n+m+1})\{(2n+2m+1)p_{3_j}q_{3_j}^{2n+2m}\}\{(2k+1)p_{4_j}q_{4_j}^{2k}\}(q_{5_j}^{2k+1})$$
$$\times (q_{6_j}^{4k+1})$$

$$+ (q_{1_j}^{n+m+1})(p_{2_j})\{(n+m+1)p_{3_j}q_{3_j}^{n+m}\}\{(2k+1)p_{4_j}q_{4_j}^{2k}\}(q_{5_j}^{2k+1})$$
$$\times (q_{6_j}^{4k+1})$$

$$+ (p_{1_j})\{(n+m+1)p_{2_j}q_{2_j}^{n+m}\}(q_{3_j}^{2k+1})(q_{4_j}^{4k+1})\{(6k+1)p_{5_j}q_{5_j}^{6k}\}$$
$$\times (q_{6_j}^{2k+1})$$

$$+ (p_{1_j})(q_{2_j}^{n+m+1})\{(2n+2m+1)p_{3_j}q_{3_j}^{2n+2m}\}(q_{4_j}^{2k+1})\{(4k+1)p_{5_j}q_{5_j}^{4k}\}$$
$$\times (q_{6_j}^{2k+1})$$

$$+ (q_{1_j}^{n+m+1})(p_{2_j})\{(n+m+1)p_{3_j}q_{3_j}^{n+m}\}(q_{4_j}^{2k+1})\{(4k+1)p_{5_j}q_{5_j}^{4k}\}$$
$$\times (q_{6_j}^{2k+1})$$

$$+ (p_{1_j})(q_{2_j}^{n+m+1})(q_{3_j}^{2n+2m+1})\{(3n+3m+1)p_{4_j}q_{4_j}^{3n+3m}\}$$
$$\{(2k+1)p_{5_j}q_{5_j}^{2k}\}(q_{6_j}^{2k+1})$$

$$+ (q_{1_j}^{n+m+1})(p_{2_j})(q_{3_j}^{n+m+1})\{(2n+2m+1)p_{4_j}q_{4_j}^{2n+2m}\}\{(2k+1)p_{5_j}q_{5_j}^{2k}\}$$
$$\times (q_{6_j}^{2k+1})$$

$$+ (q_{1_j}{}^{2n+2m+1})(q_{2_j}{}^{n+m+1})(p_{3_j})\left\{(n+m+1)p_{4_j}q_{4_j}{}^{n+m}\right\}\left\{(2k+1)p_{5_j}q_{5_j}{}^{2k}\right\}$$

$$\times (q_{6_j}{}^{2k+1})$$

$$+ (p_{1_j})\left\{(n+m+1)p_{2_j}q_{2_j}{}^{n+m}\right\}(q_{3_j}{}^{2k+1})(q_{4_j}{}^{4k+1})(q_{5_j}{}^{6k+1})$$

$$\left\{(8k+1)p_{6_j}q_{6_j}{}^{8k}\right\}$$

$$+ (p_{1_j})(q_{2_j}{}^{n+m+1})\left\{(2n+2m+1)p_{3_j}q_{3_j}{}^{2n+2m}\right\}(q_{4_j}{}^{2k+1})(q_{5_j}{}^{4k+1})$$

$$\left\{(6k+1)p_{6_j}q_{6_j}{}^{6k}\right\}$$

$$+ (q_{1_j}{}^{n+m+1})(p_{2_j})\left\{(n+m+1)p_{3_j}q_{3_j}{}^{n+m}\right\}q_{4_j}{}^{2k+1})(q_{5_j}{}^{4k+1})$$

$$\left\{(6k+1)p_{6_j}q_{6_j}{}^{6k}\right\}$$

$$+ (p_{1_j})(q_{2_j}{}^{n+m+1})(q_{3_j}{}^{2n+2m+1})\left\{(3n+3m+1)p_{4_j}q_{4_j}{}^{3n+3m}\right\}(q_{5_j}{}^{2k+1})$$

$$\left\{(4k+1)p_{6_j}q_{6_j}{}^{4k}\right\}$$

$$+ (q_{1_j}{}^{n+m+1})(p_{2_j})(q_{3_j}{}^{n+m+1})\left\{(2n+2m+1)p_{4_j}q_{4_j}{}^{2n+2m}\right\}(q_{5_j}{}^{2k+1})$$

$$\left\{(4k+1)p_{6_j}q_{6_j}{}^{4k}\right\}$$

$$+ (q_{1_j}{}^{2n+2m+m})(q_{2_j}{}^{n+m+1})(p_{3_j})\left\{(n+m+1)p_{4_j}q_{4_j}{}^{n+m}\right\}(q_{5_j}{}^{2k+1})$$

$$\left\{(4k+1)p_{6_j}q_{6_j}{}^{4k}\right\}$$

$$+ (p_{1_j})(q_{2_j}^{\,n+m+1})(q_{3_j}^{\,2n+2m+1})(q_{4_j}^{\,3n+3m+1})\{(4n+4m+1)p_{5_j}q_{5_j}^{\,4n+4m}\}$$
$$\{(2k+1)p_{6_j}q_{6_j}^{\,2k}\}$$

$$+ (q_{1_j}^{\,n+m+1})(p_{2_j})(q_{3_j}^{\,n+m+1})(q_{4_j}^{\,2n+2m+1})\{(3n+3m+1)p_{5_j}q_{5_j}^{\,3n+3m}\}$$
$$\{(2k+1)p_{6_j}q_{6_j}^{\,2k}\}$$

$$+ (q_{1_j}^{\,2n+2m+1})(q_{2_j}^{\,n+m+1})(p_{3_j})(q_{4_j}^{\,n+m+1})\{(2n+2m+1)p_{5_j}q_{5_j}^{\,2n+2m}\}$$
$$\{(2k+1)p_{6_j}q_{6_j}^{\,2k}\}$$

$$+ (q_{1j}^{\,3n+3m+1})(q_{2j}^{\,2n+2m+1})(q_{3j}^{\,n+m+1})(p_{4j})\{(n+m+1)\ p_{5j}q_{5j}^{\,n+m}\}$$
$$\{(2k+1)p_{6j}q_{6j}^{\,2k}\}\Big],$$

and

$P'(s=2;\ I\geq I_j) =$

$$(\tfrac{\tau}{\delta t})\Big[(p_{1j})\{(n+m+1)p_{2j}q_{2j}^{n+m}\}(q_{3j}^{2k+1})(q_{4j}^{4k+1})(q_{5j}^{6k+1})(q_{6j}^{8k+1})$$

$$+ (p_{1j})(q_{2j}^{n+m+1})\{(2n+2m+1)p_{3j}q_{3j}^{2n+2m}\}(q_{4j}^{2k+1})(q_{5j}^{4k+1})(q_{6j}^{6k+1})$$

$$+ (q_{1j}^{n+m+1})(p_{2j})\{(n+m+1)p_{3j}q_{3j}^{n+m}\}(q_{4j}^{2k+1})(q_{5j}^{4k+1})(q_{6j}^{6k+1})$$

$$+ (p_{1j})(q_{2j}^{n+m+1})(q_{3j}^{2n+2m+1})\{(3n+3m+1)p_{4j}q_{4j}^{3n+3m}\}(q_{5j}^{2k+1})(q_{6j}^{4k+1})$$

$$+ (q_{1j}^{n+m+1})(p_{2j})(q_{3j}^{n+m+1})\{(2n+2m+1)p_{4j}q_{4j}^{2n+2m}\}(q_{5j}^{2k+1})(q_{6j}^{4k+1})$$

$$+ (q_{1j}^{2n+2m+1})(q_{2j}^{n+m+1})(p_{3j})\{(n+m+1)p_{4j}q_{4j}^{n+m}\}(q_{5j}^{2k+1})(q_{6j}^{4k+1})$$

$$+ (p_{1j})(q_{2j}^{n+m+1})(q_{3j}^{2n+2m+1})(q_{4j}^{3n+3m+1})\{(4n+4m+1)p_{5j}q_{5j}^{4n+4m}\}(q_{6j}^{2k+1})$$

$$+ (q_{1j}^{n+m+1})(p_{2j})(q_{3j}^{n+m+1})(q_{4j}^{2n+2m+1})\{(3n+3m+1)p_{5j}q_{5j}^{3n+3m}\}(q_{6j}^{2k+1})$$

$$+ (q_{1j}^{2n+2m+1})(q_{2j}^{n+m+1})(p_{3j})(q_{4j}^{n+m+1})\{(2n+2m+1)p_{5j}q_{5j}^{2n+2m}\}(q_{6j}^{2k+1})$$

$$+ (q_{1j}^{3n+3m+1})(q_{2j}^{2n+2m+1})(q_{3j}^{n+m+1})(p_{4j})\{(n+m+1)p_{5j}q_{5j}^{n+m}\}(q_{6j}^{2k+1})$$

$$+ (p_{1j})(q_{2j}^{n+m+1})(q_{3j}^{2n+2m+1})(q_{4j}^{3n+3m+1})(q_{5j}^{4n+4m+1})\{(5n+5m+1)p_{6j}q_{6j}^{5n+5m}\}$$

$$+ (q_{1j}^{n+m+1})(p_{2j})(q_{3j}^{n+m+1})(q_{4j}^{2n+2m+1})(q_{5j}^{3n+3m+1})\{(4n+4m+1)p_{6j}q_{6j}^{4n+4m}\}$$

$$+ (q_{1j}^{2n+2m+1})(q_{2j}^{n+m+1})(p_{3j})(q_{4j}^{n+m+1})(q_{5j}^{2n+2m+1})\{(3n+3m+1)p_{6j}q_{6j}^{3n+3m}\}$$

$$+ (q_{1j}^{3n+3m+1})(q_{2j}^{2n+2m+1})(q_{3j}^{n+m+1})(p_{4j})(q_{5j}^{n+m+1})\{(2n+2m+1)p_{6j}q_{6j}^{2n+2m}\}$$

$$+ (q_{1j}^{4n+4m+1})(q_{2j}^{3n+3m+1})(q_{3j}^{2n+2m+1})(q_{4j}^{n+m+1})(p_{5j})\{(n+m+1)p_{6j}q_{6j}^{n+m}\}\Big]$$

The obvious reduction is made for λ or more <u>consecutive</u> marks on six ping cycles.

It should be noted that the multiplicative factor $(\tau/\delta t)$, representing the number of resolution intervals per ping cycle, is slightly large and should be reduced by about (3n+3m) for a six ping cycle history due to truncation at the display boundaries.

6. EXTENSION TO INCLUDE HIGHER MARKING DENSITIES

In previous sections the assumption has been made that the marking density (for a given intensity of marks) is low enough to preclude consideration of more than one mark in the range interval determined by range rate and rate of change of range rate restrictions on any ping cycle. A marking density of 0.8 has been reported[5] as furnishing the best detectability for 7 intensity levels and a 6 ping cycle history. For a 3 ping cycle history and 7 intensity levels an optimum marking density of 0.2 is reported[5]. Even for the miss distance considered earlier for zero range rates, the probability of not getting at least two marks at the lowest non-zero intensity is about 10^{-6} for the 6 ping cycle history with 0.8 marking density and about 0.74 for the 3 ping cycle history and 0.2 marking density. For reasonable range rates and intensities, the assumption made in previous sections is clearly invalid for the above cases.

The difficulty in extending the earlier treatment to include higher marking densities stems from the lack of independence between "traces" formed by joining possible marks on successive ping cycles. Consider, for example, a ν-ping cycle history and a range position corresponding to the rth resolution interval on the νth ping cycle, where $r = 1,2,\ldots,N$ (there being N range resolution intervals for each ping cycle of each beam of the display). If a range rate capability of +n to -m resolution intervals per ping cycle is allowed, then a possible trace may consist of a line joining the rth resolution interval of the νth ping cycle, any one of (n+m+1) resolution intervals of the $(\nu-1)$th ping cycle, any one of (n+m+1) resolution intervals of the $(\nu-2)$th ping cycle, etc. There are thus $(n+m+1)^{\nu-1}$ possible traces which have one or more [$(\nu-1)$maximum] resolution intervals in common. Also, traces related to the rth resolution interval of the νth ping cycle will have resolution intervals in common with traces related to other resolution intervals of the νth ping cycle.

A general expression will be sought for the probability that at least one trace of a ν-ping cycle history is marked according to a prescribed criterion [e.g., a mark (of the required intensity) on λ or more of the ν ping cycles]. This expression will be construed as the probability of at least one "false alarm". Physical arguments will then be discussed which allow reduction of the general expression to a more tractable form.

Consider again a ν-ping cycle history and a range position corresponding to the rth resolution interval of the νth ping cycle. Let P_r^i represent the probability that the ith trace related to the rth resolution interval satisfies the prescribed detection criterion, where $i = 1,2,..,(n+m+1)^{\nu-1}$. The probability that at least one of the $N' = (n+m+1)^{\nu-1}$ traces satisfies the detection criterion will be designated P_r and is given by[1]

$$P_r = \sum_{i=1}^{N'} P_r^i - \sum_{(i,j)} (P_r^i P_r^j) + \sum_{(i,j,k)} (P_r^i P_r^j P_r^k)$$

$$- \ldots + (P_r^i P_r^j P_r^k \ldots P_r^{N'}) \quad . \tag{1}$$

The quantity $(P_r^k P_r^j)$ in the second sum represents the probability of the joint event consisting of trace i <u>and</u> trace j satisfying the detection criterion simultaneously; the sum extends over all pairs drawn from N'. Similarly, the quantity $(P_r^i P_r^j P_r^k)$ represents the joint event of trace i, trace j and trace k while the sum extends over all triplets from N'.

Now P_r represents the probability of at least one "successful" trace from among the N' associated with the rth resolution interval of the νth ping cycle. There are N such resolution intervals to each of which we can ascribe a probability of at least one successful trace. The probability that at least one trace of a complete ν-ping cycle history is marked according to a prescribed criterion may be written as

$$P = \sum_{r=1}^{N} P_r - \sum_{r,s} (P_r P_s) + \sum_{r,s,u} (P_r P_s P_u) - \ldots + (P_r P_s P_u \ldots P_N). \tag{2}$$

The quantity $(P_r P_s)$ represents the probability of the joint event of at least one successful trace associated with the rth resolution interval and at least one associated with the sth resolution interval. The second sum extends over all pairs of resolution intervals of the νth ping cycle. The third and higher order sums refer to combinations of the 1,2,...,N subscripts taken 3,4,...,N at a time.

In the most general case with $N, N' \gg 1$ and a non-uniform but high noise marking density over the display, the above probability statement presents a most unpalatable computation problem. With reasonable restrictions, however, approximations to the above expression can be computed much more easily and with acceptable accuracy.

Consider first the restriction of uniform noise marking density over a ν-ping cycle history. This implies that the probability of a noise mark in any resolution interval of any of the ν-ping cycles is constant. This in turn implies all the P_r^i are equal for a given marking criterion and the first sum in Equation (1) reduces to $N'P_r^i$. A restriction may also be placed on the number of pairs it is necessary to include in the second sum of Equation (1) for any required degree of accuracy. Figure 1 will aid in discussing this restriction. The difference in the

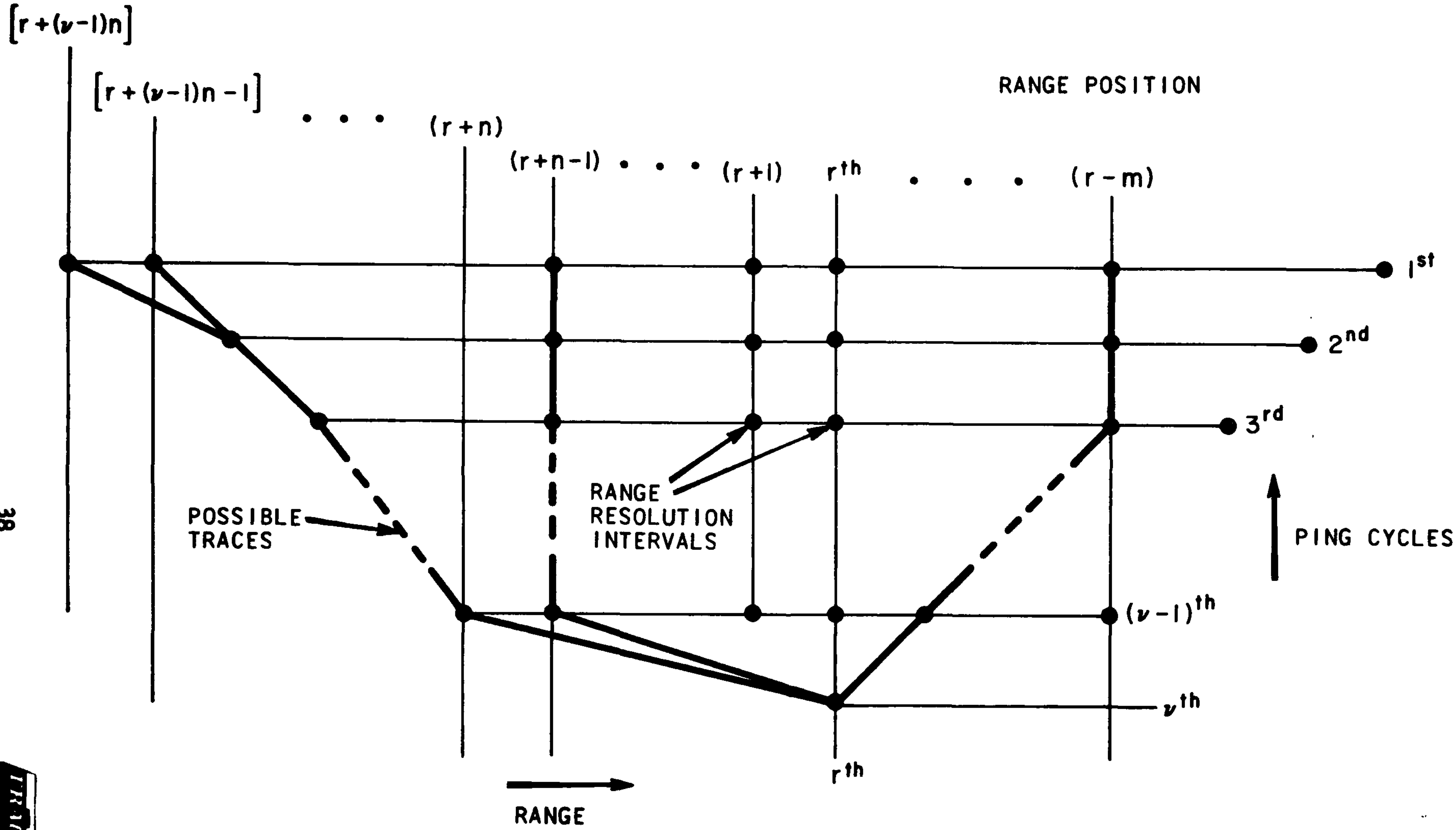

FIG. 1 POSSIBLE TRACES RESULTING FROM MARKING A ν-PING CYCLE HISTORY ACCORDING TO A REQUIRED "DETECTION" CRITERION.

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probability of a single trace and the probability of a pair of traces such as the left-most pair of Fig. 1 is the probability of marking one additional resolution interval (or not marking, depending upon the "detection" criterion). For high marking densities, such terms in the second sum of Equation (1) are of the same order of magnitude as terms of the first sum and must not be ignored. Thus, if a pair of traces share (ν-1) resolution intervals, the probability of successfully marking that pair may be as large as the product of the probability of successfully marking a single trace and the probability of marking a single resolution interval. Such terms would be retained in the second sum of Equation (1) and may be represented by $P_r^i\, p$, where p is the probability of marking a single resolution interval and is equivalent to the marking density (assumed uniform) expressed as a fraction of the total number of resolution intervals. In general, pairs will occur which share (ν-1), (ν-2), ..., 1 resotion intervals and will contribute terms in Equation (1) of order $P_r^i\, p$, $P_r^i\, p^2$, ... $P_r^i\, p^{(\nu-1)}$, respectively. The quantity p^j in these terms may be used to determine which pairs must be retained in the second sum of Equation (1) for a given marking density and required accuracy.

The argument given above can be extended to the third and higher sums of Equation (1). For traces which share (ν-1), (ν-2), ..., 1 resolution intervals, terms of the third sum are at most of order $P_r^i\, p^2$, $P_r^i\, p^3$, ..., $P_r^i\, p^{(\nu-1)2}$, respectively; terms of the fourth sum are at most of order $P_r^i\, p^3$, $P_r^i\, p^4$, ..., $P_r^i\, p^{(\nu-1)3}$, respectively. For a given marking density and required accuracy, both the number of terms of a particular sum and the number of sums may be considerably lower than the numbers indicated in the general expression of Equation (1).

An examination of the second and higher order sums of Equation (2) reveals the possibility of further reduction in computational complexity. The terms of the second sum, for example, are independent except in cases where a trace ending on the rth resolution interval of the νth ping cycle overlaps a trace ending on the sth resolution interval. Once a range rate capability has been specified, a definite overlap region for a ν-ping cycle history exists for terms in each of the sums of Equation (2). Outside the overlap regions the terms of the second and higher sums will likely be negligible compared to corresponding terms within the overlap regions and can be ignored. Their order can be examined with respect to marking density, "detection" criterion, and accuracy requirements. The number of terms of each sum and the number of sums which must be considered in Equation (2) should be significantly lower than the numbers indicated in the general expression.

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13. ABSTRACT

False alarm and detection probability statements are presented for various display marking criteria. These statements are given in terms of the marking probabilities for single range resolution intervals of the display. Intensity and pattern of marks are included in the criteria and a procedure is outlined for obtaining single range resolution interval probabilities from signal processor curves in the case of a matched display. Both zero and non-zero range rate targets are considered.

The case of a low noise marking density is considered in some detail. Physical arguments are then presented for reducing to a more tractable approximate form the expressions for high marking densities. The arguments lead to estimates of the error introduced by use of approximate or other than exact expressions.

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